

Question 5**Problem:**

Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 3x + 1$$

on the interval $[-2, 2]$.

- (a) Find the critical points of $f(x)$ in the interval.
- (b) Evaluate $f(x)$ at the endpoints and critical points.
- (c) Identify the absolute maximum and minimum values.

Question 5 - Solution

We are given:

$$f(x) = x^3 - 3x + 1$$

(a) Find critical points:

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x - 1)(x + 1)$$

Set derivative to zero:

$$f'(x) = 0 \Rightarrow x = \pm 1$$

Critical points: $x = -1, x = 1$

(b) Evaluate function:

$$f(-2) = (-2)^3 - 3(-2) + 1 = -8 + 6 + 1 = -1$$

$$f(-1) = (-1)^3 - 3(-1) + 1 = -1 + 3 + 1 = 3$$

$$f(1) = 1 - 3 + 1 = -1$$

$$f(2) = 8 - 6 + 1 = 3$$

(c) Conclusion:

• Absolute maximum: $f(-1) = f(2) = 3$

• Absolute minimum: $f(-2) = f(1) = -1$

Graph of $f(x) = x^3 - 3x + 1$ on $[-2, 2]$:

