

Question 4**Problem:**

Find the absolute maximum and minimum values of the function

$$f(x) = x^2 \ln(x)$$

on the interval $[1, 4]$.

- (a) Find the critical points of $f(x)$ in the interval.
- (b) Evaluate $f(x)$ at the endpoints and critical points.
- (c) Determine the absolute maximum and minimum values.

Question 4 - Solution

We are given:

$$f(x) = x^2 \ln(x)$$

(a) Find critical points:

Use product rule:

$$f'(x) = \frac{d}{dx}[x^2] \cdot \ln(x) + x^2 \cdot \frac{d}{dx}[\ln(x)] = 2x \ln(x) + x$$

Set derivative to zero:

$$f'(x) = 2x \ln(x) + x = x(2 \ln(x) + 1) = 0 \Rightarrow 2 \ln(x) + 1 = 0 \Rightarrow \ln(x) = -\frac{1}{2} \Rightarrow x = e^{-1/2}$$

Since $e^{-1/2} \approx 0.606 \notin [1, 4]$, there are no critical points in the interval.

(b) Evaluate at endpoints:

$$f(1) = 1^2 \ln(1) = 0, \quad f(4) = 16 \ln(4) = 16 \cdot \ln(2^2) = 16 \cdot 2 \ln(2) \approx 16 \cdot 1.386 = 22.18$$

(c) Conclusion:

- Absolute minimum: $f(1) = 0$
- Absolute maximum: $f(4) \approx 22.18$

Graph of $f(x) = x^2 \ln(x)$ on $[1, 4]$:

