

Question 3**Problem:**

Find the absolute maximum and minimum values of the function

$$f(x) = \frac{x}{x^2 + 1}$$

on the interval $[-2, 2]$.

- (a) Find the critical points of $f(x)$ in the interval.
- (b) Evaluate $f(x)$ at the endpoints and at critical points.
- (c) Determine the absolute maximum and minimum values.

Question 3 - Solution

We are given:

$$f(x) = \frac{x}{x^2 + 1}$$

(a) Find critical points:

Use quotient rule:

$$f'(x) = \frac{(x^2 + 1)(1) - x(2x)}{(x^2 + 1)^2} = \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}$$

Set $f'(x) = 0$:

$$\frac{1 - x^2}{(x^2 + 1)^2} = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x = \pm 1$$

So, critical points: $x = -1$, $x = 1$

(b) Evaluate at endpoints and critical points:

$$f(-2) = \frac{-2}{4 + 1} = -\frac{2}{5}, \quad f(-1) = \frac{-1}{1 + 1} = -\frac{1}{2}$$
$$f(1) = \frac{1}{2}, \quad f(2) = \frac{2}{5}$$

(c) Conclusion:

- Absolute minimum: $f(-1) = -\frac{1}{2}$
- Absolute maximum: $f(1) = \frac{1}{2}$

Graph of $f(x) = \frac{x}{x^2 + 1}$ on $[-2, 2]$:

