

Question 2**Problem:**

Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 3x^2 + 1$$

on the interval $[0, 4]$.

- (a) Identify all critical points of $f(x)$ in the interval.
- (b) Evaluate $f(x)$ at the endpoints and critical points.
- (c) Determine the absolute maximum and minimum values.

Question 2 - Solution

We are given:

$$f(x) = x^3 - 3x^2 + 1$$

(a) **Find critical points:**

Take the derivative:

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

Set $f'(x) = 0$:

$$3x(x - 2) = 0 \Rightarrow x = 0, x = 2$$

Both are in the interval $[0, 4]$.

(b) **Evaluate function at critical points and endpoints:**

$$f(0) = 0^3 - 3(0)^2 + 1 = 1$$

$$f(2) = 8 - 12 + 1 = -3$$

$$f(4) = 64 - 48 + 1 = 17$$

(c) **Absolute extrema:**

• Absolute minimum: $f(2) = -3$

• Absolute maximum: $f(4) = 17$

Graph of $f(x) = x^3 - 3x^2 + 1$ on $[0, 4]$:

