

Question 10**Problem:**

A box with a square base and open top must have a volume of 500 cm^3 . Find the dimensions of the box that minimize the surface area.

- (a) Express the surface area as a function of one variable.
- (b) Find the dimensions that minimize the surface area.
- (c) What is the minimum surface area?

Question 10 - Solution

Let x be the side length of the square base, and h be the height of the box.

(a) Surface area as a function of one variable:

Volume constraint:

$$x^2 h = 500 \Rightarrow h = \frac{500}{x^2}$$

Surface area (no top):

$$A(x) = x^2 + 4xh = x^2 + 4x \cdot \frac{500}{x^2} = x^2 + \frac{2000}{x}$$

(b) Minimize the surface area:

Differentiate:

$$A'(x) = 2x - \frac{2000}{x^2}$$

Set derivative to 0:

$$2x - \frac{2000}{x^2} = 0 \Rightarrow 2x^3 = 2000 \Rightarrow x^3 = 1000 \Rightarrow x = 10$$

Find h :

$$h = \frac{500}{10^2} = 5$$

Dimensions:

$$x = 10 \text{ cm}, h = 5 \text{ cm}$$

(c) Minimum surface area:

$$A = 10^2 + \frac{2000}{10} = 100 + 200 = \boxed{300 \text{ cm}^2}$$

Graph of $A(x) = x^2 + \frac{2000}{x}$:

