

Question 9**Problem:**

A farmer has 240 meters of fencing to enclose a rectangular field. One side of the rectangle lies along a river and does not require fencing.

- (a) Express the area of the field as a function of one variable.
- (b) Find the dimensions of the field that maximize the area.
- (c) What is the maximum area?

Question 9 - Solution

(a) Express the area as a function:

Let x be the length of the side perpendicular to the river. Since only three sides are fenced (two x 's and one along the length), the total fencing is:

$$2x + y = 240 \Rightarrow y = 240 - 2x$$

Area:

$$A(x) = x \cdot y = x(240 - 2x) = 240x - 2x^2$$

(b) Maximize the area:

Differentiate:

$$A'(x) = 240 - 4x$$

Set derivative to zero:

$$240 - 4x = 0 \Rightarrow x = 60$$

Check that this gives a maximum using second derivative:

$$A''(x) = -4 < 0 \Rightarrow \text{Maximum}$$

Find y :

$$y = 240 - 2(60) = 120$$

Dimensions: 60 m $\times$ 120 m

(c) Maximum area:

$$A = 60 \cdot 120 = \text{7200 m}^2$$

Graph of $A(x) = 240x - 2x^2$:

