

Question 8**Problem:**

Let $f(x) = \frac{x}{x^2+1}$ on the interval $[-3, 3]$.

- (a) Find all critical points of $f(x)$ in the interval.
- (b) Determine the absolute maximum and minimum values of $f(x)$ on the given interval.

Question 8 - Solution

(a) Find critical points:

Differentiate using the quotient rule:

$$f(x) = \frac{x}{x^2 + 1}, \quad f'(x) = \frac{(1)(x^2 + 1) - x(2x)}{(x^2 + 1)^2} = \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}$$

Set $f'(x) = 0$:

$$1 - x^2 = 0 \Rightarrow x = \pm 1$$

(b) Evaluate function at endpoints and critical points:

$$f(-3) = \frac{-3}{9+1} = -\frac{3}{10} \quad f(-1) = \frac{-1}{2} = -\frac{1}{2} \quad f(1) = \frac{1}{2} \quad f(3) = \frac{3}{10}$$

Conclusion:- Absolute maximum: $\boxed{\frac{1}{2}}$ at $x = 1$ - Absolute minimum: $\boxed{-\frac{1}{2}}$ at $x = -1$ **Graph of** $f(x) = \frac{x}{x^2+1}$ on $[-3, 3]$: