

Question 6**Problem:**

Consider the function $f(x) = x^4 - 4x^2$ on the interval $[-3, 3]$.

- (a) Find all critical points of $f(x)$ in the interval.
- (b) Determine the absolute maximum and minimum values of $f(x)$ on $[-3, 3]$.

Question 6 - Solution

(a) Find critical points:

Differentiate:

$$f'(x) = 4x^3 - 8x$$

Set derivative to zero:

$$4x^3 - 8x = 0 \Rightarrow 4x(x^2 - 2) = 0 \Rightarrow x = 0, \pm\sqrt{2}$$

Critical points in $[-3, 3]$: $x = -\sqrt{2}, 0, \sqrt{2}$ (b) Evaluate $f(x)$ at critical points and endpoints:

$$f(-3) = (-3)^4 - 4(-3)^2 = 81 - 36 = 45$$

$$f(-\sqrt{2}) = (\sqrt{2})^4 - 4(\sqrt{2})^2 = 4 - 8 = -4$$

$$f(0) = 0^4 - 4(0)^2 = 0$$

$$f(\sqrt{2}) = 4 - 8 = -4$$

$$f(3) = 81 - 36 = 45$$

Conclusion:- Absolute maximum: $\boxed{45}$ at $x = -3$ and $x = 3$ - Absolute minimum: $\boxed{-4}$ at $x = \pm\sqrt{2}$ **Graph of** $f(x) = x^4 - 4x^2$ on $[-3, 3]$: