

Question 5**Problem:**

Let $f(x) = x^3 - 3x + 1$ on the interval $[-3, 3]$.

- (a) Find all critical points of $f(x)$ on the interval.
- (b) Determine the absolute maximum and minimum values of $f(x)$ on $[-3, 3]$.

Question 5 - Solution**(a) Find critical points:**

Take the derivative:

$$f'(x) = 3x^2 - 3$$

Set derivative equal to 0:

$$3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

Critical points: $x = -1, x = 1$ **(b) Evaluate at critical points and endpoints:**

$$f(-3) = (-3)^3 - 3(-3) + 1 = -27 + 9 + 1 = -17$$

$$f(-1) = (-1)^3 - 3(-1) + 1 = -1 + 3 + 1 = 3$$

$$f(1) = 1^3 - 3(1) + 1 = 1 - 3 + 1 = -1$$

$$f(3) = 27 - 9 + 1 = 19$$

Conclusion:- Absolute maximum: $\boxed{19}$ at $x = 3$ - Absolute minimum: $\boxed{-17}$ at $x = -3$ **Graph of** $f(x) = x^3 - 3x + 1$ on $[-3, 3]$: