

Question 4**Problem:**

Let $f(x) = \frac{x}{x^2+1}$, and consider the interval $[-2, 2]$.

- (a) Find the critical points of $f(x)$ on the interval.
- (b) Determine the absolute maximum and minimum values of $f(x)$ on $[-2, 2]$.

Question 4 - Solution**(a) Find the critical points:**

We find $f'(x)$ using the quotient rule:

$$f'(x) = \frac{(x^2 + 1)(1) - x(2x)}{(x^2 + 1)^2} = \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}$$

Set the numerator equal to zero:

$$1 - x^2 = 0 \Rightarrow x = \pm 1$$

Both $x = -1$ and $x = 1$ are in the interval $[-2, 2]$.

(b) Evaluate at critical points and endpoints:

$$f(-2) = \frac{-2}{4 + 1} = \frac{-2}{5}$$

$$f(-1) = \frac{-1}{1 + 1} = \frac{-1}{2}$$

$$f(1) = \frac{1}{1 + 1} = \frac{1}{2}$$

$$f(2) = \frac{2}{4 + 1} = \frac{2}{5}$$

Conclusion:

- Maximum value: $\boxed{\frac{1}{2}}$ at $x = 1$ - Minimum value: $\boxed{-\frac{1}{2}}$ at $x = -1$

Graph of $f(x) = \frac{x}{x^2+1}$ on $[-2, 2]$:

