

Question 2**Problem:**

Let $f(x) = x^3 - 6x^2 + 9x + 2$ on the closed interval $[0, 4]$.

- (a) Find the critical points of $f(x)$ in the interval.
- (b) Determine the absolute maximum and minimum values of $f(x)$ on the interval $[0, 4]$.

Question 2 - Solution**(a) Find critical points:**

First, compute the derivative:

$$f'(x) = 3x^2 - 12x + 9$$

Set the derivative to zero:

$$3x^2 - 12x + 9 = 0 \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow (x - 1)(x - 3) = 0 \Rightarrow x = 1, x = 3$$

Both critical points are in the interval $[0, 4]$.

(b) Evaluate function at critical points and endpoints:

$$f(0) = 0 - 0 + 0 + 2 = 2$$

$$f(1) = 1 - 6 + 9 + 2 = 6$$

$$f(3) = 27 - 54 + 27 + 2 = 2$$

$$f(4) = 64 - 96 + 36 + 2 = 6$$

Conclusion:

- Maximum value: $\boxed{6}$ at $x = 1$ and $x = 4$ - Minimum value: $\boxed{2}$ at $x = 0$ and $x = 3$

Graph of the function on $[0, 4]$:

