

Question 10**Problem:**

Consider the function:

$$f(x) = x^4 - 4x^3 + 6x^2 + 1$$

- (a) Find all critical points of $f(x)$.
- (b) Classify each critical point using the First Derivative Test.
- (c) Determine the function value at each critical point.

Question 10 - Solution

(a) Find critical points:

First, compute the derivative:

$$f'(x) = 4x^3 - 12x^2 + 12x$$

Factor:

$$f'(x) = 4x(x^2 - 3x + 3)$$

Set derivative to zero:

$$4x(x^2 - 3x + 3) = 0 \Rightarrow x = 0 \quad (\text{real root})$$

Solve $x^2 - 3x + 3 = 0$:

$$\text{Discriminant: } D = (-3)^2 - 4(1)(3) = 9 - 12 = -3 \Rightarrow \text{no real roots}$$

Critical point: $x = 0$

(b) **First Derivative Test:** Test sign of $f'(x)$ around $x = 0$:

For $x < 0$: try $x = -1$

$$f'(-1) = 4(-1)^3 - 12(-1)^2 + 12(-1) = -4 - 12 - 12 = -28 < 0$$

For $x > 0$: try $x = 1$

$$f'(1) = 4(1)^3 - 12(1)^2 + 12(1) = 4 - 12 + 12 = 4 > 0$$

Since $f'(x)$ changes from negative to positive at $x = 0$, there is a local minimum.

(c) **Function value at critical point:**

$$f(0) = 0^4 - 4(0)^3 + 6(0)^2 + 1 = 1$$

Local minimum at: $(0, 1)$

Graph of the function:

