

Question 9**Problem:**

Let the function be defined as:

$$f(x) = x^3 - 3x^2 - 9x + 5$$

- (a) Find all critical points of $f(x)$.
- (b) Use the First Derivative Test to classify each critical point as a local maximum or local minimum.
- (c) Determine the function value at each critical point.

Question 9 - Solution

(a) Find critical points:

Differentiate:

$$f'(x) = 3x^2 - 6x - 9$$

Set $f'(x) = 0$:

$$3x^2 - 6x - 9 = 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x - 3)(x + 1) = 0 \Rightarrow x = -1, 3$$

Critical points: $x = -1, x = 3$

(b) First Derivative Test:

Interval $(-\infty, -1)$: Choose $x = -2$, $f'(-2) = 3(4) + 12 - 9 = 15 > 0$ Interval $(-1, 3)$: Choose $x = 0$, $f'(0) = -9 < 0$ Interval $(3, \infty)$: Choose $x = 4$, $f'(4) = 3(16) - 24 - 9 = 15 > 0$ So: At $x = -1$, f' changes from positive to negative $\rightarrow$ local maximum At $x = 3$, f' changes from negative to positive $\rightarrow$ local minimum

(c) Function values:

$$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 5 = -1 - 3 + 9 + 5 = 10$$

$$f(3) = 27 - 27 - 27 + 5 = -22$$

Local maximum: $f(-1) = 10$ Local minimum: $f(3) = -22$

Graph of the function:

