

Question 7**Problem:**

A company's profit function (in thousands of dollars) from selling x hundred units of a product is given by:

$$P(x) = -x^3 + 6x^2 + 60$$

- (a) Find all critical points of $P(x)$.
- (b) Determine whether each critical point is a local maximum or minimum using the First Derivative Test.
- (c) Interpret your result in the context of the company's production.

Question 7 - Solution

(a) Find critical points:

Take the derivative:

$$P'(x) = -3x^2 + 12x$$

Set $P'(x) = 0$:

$$-3x^2 + 12x = 0 \Rightarrow x(-3x + 12) = 0 \Rightarrow x = 0 \text{ or } x = 4$$

Critical points: $x = 0$, $x = 4$

(b) First Derivative Test:

Check intervals: For $x < 0$: not relevant (negative production)Between $0 < x < 4$: pick $x = 2$, $P'(2) = -3(4) + 12(2) = -12 + 24 = 12 > 0$ For $x > 4$: pick $x = 5$, $P'(5) = -3(25) + 12(5) = -75 + 60 = -15 < 0$

Conclusion: $P(x)$ increases before $x = 4$, decreases after $\rightarrow$ local max at $x = 4$
 $x = 0$ is a stationary point, but no increase before it (since domain is $x \geq 0$)

(c) Interpretation:

Producing 400 units ($x = 4$) yields the maximum profit. Profit decreases beyond 400 units, and no benefit is gained producing less than 400.

Maximum profit occurs when the company produces 400 units.

Profit Function Graph:

