

Question 6**Problem:**Let $f(x) = \frac{x^3}{3} - 2x^2 + x + 5$.

- (a) Find all critical points of $f(x)$.
- (b) Use the First Derivative Test to classify each critical point as a local maximum, local minimum, or neither.

Question 6 - Solution

We are given:

$$f(x) = \frac{x^3}{3} - 2x^2 + x + 5$$

(a) Find the critical points:

Take the first derivative:

$$f'(x) = x^2 - 4x + 1$$

Set $f'(x) = 0$:

$$x^2 - 4x + 1 = 0 \Rightarrow x = 2 \pm \sqrt{3}$$

Critical points: $x = 2 + \sqrt{3}$, $x = 2 - \sqrt{3}$

(b) First Derivative Test:

Evaluate $f'(x)$ on intervals around the critical points:

Interval $(-\infty, 2 - \sqrt{3})$: pick $x = 0$, $f'(0) = 1 > 0$

Interval $(2 - \sqrt{3}, 2 + \sqrt{3})$: pick $x = 2$, $f'(2) = -3 < 0$

Interval $(2 + \sqrt{3}, \infty)$: pick $x = 4$, $f'(4) = 1 > 0$

Conclusion:

$f'(x)$ changes from positive to negative at $x = 2 - \sqrt{3} \rightarrow$ local maximum $f'(x)$ changes from negative to positive at $x = 2 + \sqrt{3} \rightarrow$ local minimum

Local max at $x = 2 - \sqrt{3}$
Local min at $x = 2 + \sqrt{3}$

