

Question 5**Problem**

Let

$$f(x) = x^4 - 4x^3 + 6x^2.$$

- (a) Find the critical points of $f(x)$.
- (b) Classify each critical point as a local maximum, local minimum, or neither.

Question 5 - Solution

We are given

$$f(x) = x^4 - 4x^3 + 6x^2.$$

(a) Find the critical points.

Differentiate:

$$f'(x) = 4x^3 - 12x^2 + 12x.$$

Factor:

$$f'(x) = 4x(x^2 - 3x + 3).$$

Set $f'(x) = 0$:

$$4x(x^2 - 3x + 3) = 0.$$

This gives $x = 0$, or $x^2 - 3x + 3 = 0$. The quadratic has discriminant

$$(-3)^2 - 4(1)(3) = 9 - 12 = -3 < 0,$$

so it has no real solutions. Therefore,

Only critical point: $x = 0$.

(b) Classify the critical point.

Compute the second derivative:

$$f''(x) = 12x^2 - 24x + 12.$$

Evaluate at $x = 0$:

$$f''(0) = 12 > 0,$$

so $f(x)$ has a local minimum at $x = 0$. Also $f(0) = 0$, so the minimum point is $(0, 0)$.

Conclusion

Local minimum at $x = 0$ (point $(0, 0)$)

Graph of $f(x)$ with the critical point marked

