

Question 4**Problem**

A company models its revenue $R(x)$, in thousands of dollars, as a function of the number of items x (in hundreds) it produces and sells:

$$R(x) = -x^3 + 12x^2 + 9x.$$

- (a) Find all critical points of $R(x)$.
- (b) Use the First Derivative Test to classify each critical point as a local maximum, local minimum, or neither.
- (c) How many items should be produced to maximize revenue?

Question 4 - Solution

We are given

$$R(x) = -x^3 + 12x^2 + 9x.$$

(a) Find the derivative:

$$R'(x) = -3x^2 + 24x + 9.$$

Set $R'(x) = 0$:

$$-3x^2 + 24x + 9 = 0 \Rightarrow x^2 - 8x - 3 = 0.$$

Using the quadratic formula:

$$x = \frac{8 \pm \sqrt{64 + 12}}{2} = \frac{8 \pm \sqrt{76}}{2} = 4 \pm \sqrt{19}.$$

Critical points: $x = 4 - \sqrt{19}$, $4 + \sqrt{19}$

(b) First Derivative Test

Since $R'(x)$ is a downward-opening parabola, it is negative outside its roots and positive between them.

- At $x = 4 - \sqrt{19}$, $R'(x)$ changes from negative to positive, so $R(x)$ has a local minimum.
- At $x = 4 + \sqrt{19}$, $R'(x)$ changes from positive to negative, so $R(x)$ has a local maximum.

(c) Maximizing revenue

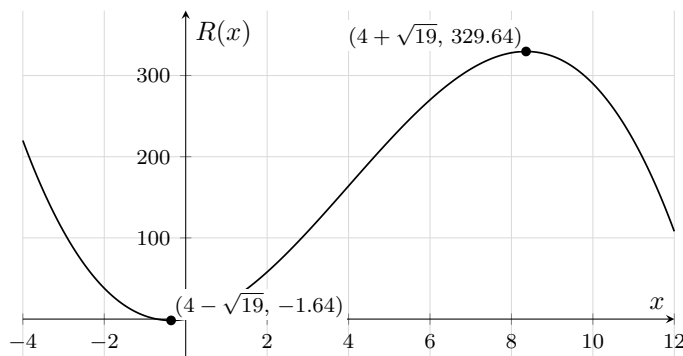
Revenue is maximized at

$$x = 4 + \sqrt{19} \approx 8.36.$$

Since x is measured in hundreds of items, this corresponds to

836 items.

Graph of $R(x)$ with critical points marked



x : hundreds of items; $R(x)$: thousands of dollars.
Point heights are rounded to two decimal places.