

Question 2

A company models its profit $P(x)$, in thousands of dollars, as a function of the number of units x (in hundreds) it produces and sells:

$$P(x) = -2x^3 + 15x^2 + 36x.$$

- (a) Find the critical points of the function.
- (b) Use the First Derivative Test to classify each critical point as a local maximum, minimum, or neither.
- (c) How many units should be produced to maximize profit?

Question 2 - Solution

We are given

$$P(x) = -2x^3 + 15x^2 + 36x.$$

(a) Find $P'(x)$:

$$P'(x) = -6x^2 + 30x + 36.$$

Set $P'(x) = 0$ to find critical points:

$$-6x^2 + 30x + 36 = 0 \Rightarrow x^2 - 5x - 6 = 0 \Rightarrow (x - 6)(x + 1) = 0.$$

Critical points: $x = -1, 6$

(b) First Derivative Test:

Evaluate the sign of $P'(x)$ on intervals determined by the critical points.

- On $(-\infty, -1)$, pick $x = -2$:

$$P'(-2) = -6(4) + 30(-2) + 36 = -24 - 60 + 36 = -48 < 0.$$

- On $(-1, 6)$, pick $x = 0$:

$$P'(0) = 36 > 0.$$

- On $(6, \infty)$, pick $x = 7$:

$$P'(7) = -6(49) + 30(7) + 36 = -294 + 210 + 36 = -48 < 0.$$

Conclusion:

- At $x = -1$, $P'(x)$ changes from negative to positive, so P has a local minimum.
- At $x = 6$, $P'(x)$ changes from positive to negative, so P has a local maximum.

(c) Maximizing profit:

Since $x = 6$ is a local maximum, the profit is maximized at $x = 6$ (hundreds of units), which is

$$6 \times 100 = \boxed{600 \text{ units}}.$$

Compute the maximum profit:

$$P(6) = -2(6)^3 + 15(6)^2 + 36(6) = -432 + 540 + 216 = \boxed{324}.$$

This represents $\boxed{\$324,000}$ since P is in thousands of dollars.

Graph of $P(x)$ with critical points marked

