

Question 1

Let

$$f(x) = x^3 - 6x^2 + 9x + 1.$$

- (a) Find all critical points of $f(x)$.
- (b) Classify each critical point as a local minimum, local maximum, or neither using the First Derivative Test.

Question 1 - Solution

We are given

$$f(x) = x^3 - 6x^2 + 9x + 1.$$

(a) Find $f'(x)$:

$$f'(x) = 3x^2 - 12x + 9.$$

Set $f'(x) = 0$ to find critical points:

$$3x^2 - 12x + 9 = 0 \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow (x - 1)(x - 3) = 0.$$

Critical points: $x = 1, 3$

(b) First Derivative Test:

- Choose test values around $x = 1$ and $x = 3$.
- Use $f'(x) = 3x^2 - 12x + 9$.

Evaluate the sign of $f'(x)$:

- On $(-\infty, 1)$, pick $x = 0$:

$$f'(0) = 9 > 0.$$

- On $(1, 3)$, pick $x = 2$:

$$f'(2) = 3(4) - 24 + 9 = -3 < 0.$$

- On $(3, \infty)$, pick $x = 4$:

$$f'(4) = 48 - 48 + 9 = 9 > 0.$$

Conclusion:

- At $x = 1$, $f'(x)$ changes from positive to negative, so f has a local maximum.
- At $x = 3$, $f'(x)$ changes from negative to positive, so f has a local minimum.

Answer:

- Local maximum at $x = 1$ with $f(1) = 5$
- Local minimum at $x = 3$ with $f(3) = 1$

Graph of $f(x)$ with critical points marked

