

Question 10

A company manufactures and sells a product. The weekly revenue and cost functions (in thousands of dollars) are:

$$R(x) = 500x - 5x^2, \quad C(x) = 200x + 500$$

- (a) Determine the profit function $P(x)$.
- (b) Find the production level x that maximizes profit.
- (c) Compute the maximum profit.

Question 10 - Solution**(a) Profit Function:**

$$P(x) = R(x) - C(x) = (500x - 5x^2) - (200x + 500) = -5x^2 + 300x - 500$$

(b) Maximize Profit:

$$P'(x) = -10x + 300$$

Set $P'(x) = 0$:

$$-10x + 300 = 0 \quad \Rightarrow \quad x = 30$$

Check the second derivative:

$$P''(x) = -10 < 0$$

Thus, profit is maximized when $x = 30$.**(c) Maximum Profit:**

$$P(30) = -5(30)^2 + 300(30) - 500 = -4500 + 9000 - 500 = 4000$$

Maximum profit occurs at $x = 30$ units, $P_{\max} = 4000$ thousand dollars