

Question 9

The demand for a product is modeled by

$$p(x) = 80 - x,$$

where $p(x)$ is the price per unit in dollars and x is the number of units sold in hundreds. The total cost of producing $100x$ units is

$$C(x) = 20x + 100 \quad \text{dollars.}$$

Require $x \geq 0$ and $p(x) \geq 0$.

- (a) Find the revenue and profit functions.
- (b) Find the production level and maximum profit in the continuous model.
- (c) Find the best whole-number quantity of units and its profit.

Question 9 - Solution**(a) Revenue and profit**

The number sold is $100x$, so revenue is $100x p(x)$:

$$R(x) = 8000x - 100x^2, \quad P(x) = -100x^2 + 7980x - 100.$$

$$P'(x) = -200x + 7980, \quad P''(x) = -200 < 0.$$

(b) Continuous maximum

The continuous maximum occurs at

$$x = \frac{7980}{200} \approx 39.900000, \quad P_{\max} \approx \$159,101.00.$$

(c) Whole units

This represents 3990.000000 units. Comparing the adjacent whole-unit counts gives 3990 units, with profit \$159,101.00. The price is nonnegative at these levels.