

Question 8

The weekly revenue (in thousands of dollars) for producing x units is:

$$R(x) = 120x - x^2$$

and the weekly cost is:

$$C(x) = 40x + 200$$

- (a) Determine the profit function $P(x)$.
- (b) Find the number of units x that maximizes profit.
- (c) Compute the maximum profit.

Question 8 - Solution**(a) Profit Function:**

$$P(x) = R(x) - C(x) = (120x - x^2) - (40x + 200) = -x^2 + 80x - 200$$

(b) Maximize Profit:

$$P'(x) = -2x + 80$$

Set $P'(x) = 0$:

$$-2x + 80 = 0 \quad \Rightarrow \quad x = 40$$

Second derivative:

$$P''(x) = -2 < 0$$

Hence, profit is maximized when $x = 40$.**(c) Maximum Profit:**

$$P(40) = -(40)^2 + 80(40) - 200 = -1600 + 3200 - 200 = 1400$$

Maximum profit occurs at $x = 40$ units, $P_{\max} = 1400$ thousand dollars