

Question 7

A company's profit $P(x)$ (in dollars) depends on the number of items x produced per week:

$$P(x) = -5x^2 + 300x - 2000$$

- (a) Determine the production level x that maximizes profit.
- (b) Compute the maximum profit.
- (c) Find the production levels x at which the profit is zero.

Question 7 - Solution**(a) Maximize Profit:**

$$P'(x) = \frac{d}{dx}(-5x^2 + 300x - 2000) = -10x + 300$$

Set $P'(x) = 0$:

$$-10x + 300 = 0 \quad \Rightarrow \quad x = 30$$

Second derivative:

$$P''(x) = -10 < 0$$

Hence, profit is maximized at $x = 30$.**(b) Maximum Profit:**

$$P(30) = -5(30)^2 + 300(30) - 2000 = -4500 + 9000 - 2000 = 2500$$

(c) Break-even Points (Profit = 0):

$$-5x^2 + 300x - 2000 = 0$$

Divide through by 5:

$$x^2 - 60x + 400 = 0$$

Solve using the quadratic formula:

$$x = \frac{60 \pm \sqrt{3600 - 1600}}{2} = \frac{60 \pm \sqrt{2000}}{2} = 30 \pm 10\sqrt{5}$$

Maximum profit occurs at $x = 30$ units, $P_{\max} = 2500$,
Profit is zero at $x = 30 - 10\sqrt{5}$, $30 + 10\sqrt{5}$.