

Question 4

The demand for a product is modeled by

$$p(x) = 100 - 2x,$$

where $p(x)$ is the price per unit in dollars and x is the number of units sold in hundreds. The total cost of producing $100x$ units is

$$C(x) = 20x + 200 \quad \text{dollars.}$$

Require $x \geq 0$ and $p(x) \geq 0$.

- (a) Find the revenue and profit functions.
- (b) Find the production level and maximum profit in the continuous model.
- (c) Find the best whole-number quantity of units and its profit.

Question 4 - Solution**(a) Revenue and profit**

The number sold is $100x$, so revenue is $100x p(x)$:

$$R(x) = 10000x - 200x^2, \quad P(x) = -200x^2 + 9980x - 200.$$

$$P'(x) = -400x + 9980, \quad P''(x) = -400 < 0.$$

(b) Continuous maximum

The continuous maximum occurs at

$$x = \frac{9980}{400} \approx 24.950000, \quad P_{\max} \approx \$124,300.50.$$

(c) Whole units

This represents 2495.000000 units. Comparing the adjacent whole-unit counts gives 2495 units, with profit \$124,300.50. The price is nonnegative at these levels.