

Question 3

A company's profit $P(x)$ (in thousands of dollars) depends on the number of units x produced per week:

$$P(x) = -2x^2 + 40x - 120$$

- (a) Determine the production level x that maximizes profit.
- (b) Compute the maximum profit.
- (c) Find the production level x at which the profit is zero.

Question 3 - Solution**(a) Maximize Profit:**

$$P'(x) = \frac{d}{dx}(-2x^2 + 40x - 120) = -4x + 40$$

Set $P'(x) = 0$:

$$-4x + 40 = 0 \quad \Rightarrow \quad x = 10$$

Second derivative:

$$P''(x) = -4 < 0$$

Hence, profit is maximized at $x = 10$.**(b) Maximum Profit:**

$$P(10) = -2(10)^2 + 40(10) - 120 = -200 + 400 - 120 = 80$$

(c) Break-even Points (Profit = 0):

$$-2x^2 + 40x - 120 = 0$$

Divide through by 2:

$$x^2 - 20x + 60 = 0$$

Solve using the quadratic formula:

$$x = \frac{20 \pm \sqrt{400 - 240}}{2} = \frac{20 \pm \sqrt{160}}{2} = 10 \pm 2\sqrt{10}$$

Maximum profit occurs at $x = 10$ units, $P_{\max} = 80$ thousand dollars,
Profit is zero at $x = 10 - 2\sqrt{10}$, $10 + 2\sqrt{10}$.