

Question 2

A company manufactures and sells x units of a product per week. The revenue (in thousands of dollars) and cost (in thousands of dollars) functions are given by:

$$R(x) = 50x - 0.5x^2, \quad C(x) = 20x + 100$$

- (a) Find the profit function $P(x)$.
- (b) Determine the number of units x that maximizes the profit.
- (c) Compute the maximum profit.

Question 2 - Solution**(a) Profit Function:**

$$P(x) = R(x) - C(x) = (50x - 0.5x^2) - (20x + 100) = -0.5x^2 + 30x - 100$$

(b) Maximize Profit:

$$P'(x) = -x + 30$$

Set $P'(x) = 0$:

$$-x + 30 = 0 \quad \Rightarrow \quad x = 30$$

Second derivative:

$$P''(x) = -1 < 0$$

Hence, profit is maximized at $x = 30$.**(c) Maximum Profit:**

$$P(30) = -0.5(30)^2 + 30(30) - 100 = -450 + 900 - 100 = 350$$

Maximum profit occurs at $x = 30$ units, $P_{\max} = 350$ thousand dollars