

Question 10

Consider the equation

$$x^3 - 3x + 1 = 0.$$

- (a) Explain why Newton's method fails with the initial guess $x_1 = 1$.
- (b) Restart with $x_1 = 0$, and compute x_2 , x_3 , and x_4 . Round the answers to four decimal places.

Question 10 - Solution

At $x_1 = 1$, $f(1) = -1$ and $f'(1) = 0$, so the Newton step is undefined. Restart at $x_1 = 0$. Newton's iteration is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f(x) = x^3 - 3x + 1, \quad f'(x) = 3x^2 - 3.$$

Keeping full precision internally gives

$$x_2 = 0.0000000000 - \frac{1.0000000000}{-3.0000000000} \approx \boxed{0.3333}$$

$$x_3 = 0.3333333333 - \frac{0.0370370370}{-2.6666666667} \approx \boxed{0.3472}$$

$$x_4 = 0.3472222222 - \frac{0.0001955804}{-2.6383101852} \approx \boxed{0.3473}$$