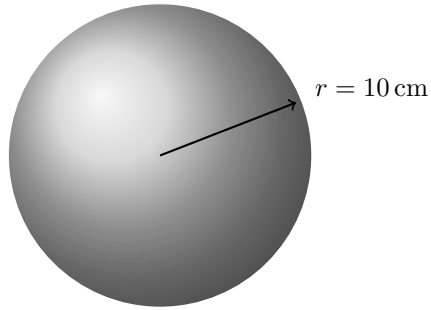


Question 8

The radius of a spherical balloon is measured to be 10 cm with a possible error of 0.05 cm.

- (a) Use differentials to estimate the maximum error in calculating the volume of the balloon.
- (b) Estimate the relative and percentage error in the volume.



Question 8 - Solution

We are given:

$$r = 10 \text{ cm}, \quad dr = 0.05 \text{ cm}$$

The volume of a sphere is:

$$V = \frac{4}{3}\pi r^3$$

Differentiate using differentials:

$$dV = \frac{d}{dr} \left(\frac{4}{3}\pi r^3 \right) \cdot dr = 4\pi r^2 \cdot dr$$

Substitute known values:

$$dV = 4\pi(10)^2 \cdot 0.05 = 4\pi \cdot 100 \cdot 0.05 = 20\pi \approx \boxed{62.83 \text{ cm}^3}$$

(a) Maximum error in volume:

$$\boxed{dV \approx 62.83 \text{ cm}^3}$$

(b) Relative and percentage error:

Actual volume:

$$V = \frac{4}{3}\pi(10)^3 = \frac{4000}{3}\pi \approx 4188.79 \text{ cm}^3$$

$$\text{Relative error} = \frac{dV}{V} \approx \frac{62.83}{4188.79} \approx 0.015$$

$$\text{Percentage error} = 0.015 \times 100 = \boxed{1.5\%}$$

Final Answers:

- Maximum error in volume: $\boxed{62.83 \text{ cm}^3}$
- Relative error: $\boxed{0.015}$
- Percentage error: $\boxed{1.5\%}$