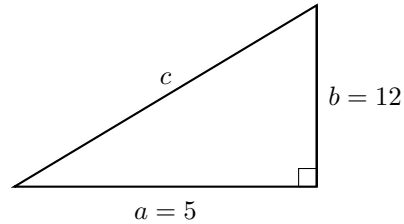


Question 7

A right triangle has one leg of fixed length 5 cm. The other leg is measured to be 12 cm with a possible measurement error of 0.1 cm.

- (a) Use differentials to approximate the maximum error in computing the hypotenuse.
- (b) Find the relative and percentage error in the hypotenuse.



Question 7 - Solution

We are given that $a = 5$ is fixed and $b = 12$ with a possible error $db = 0.1$. The hypotenuse is:

$$c = \sqrt{a^2 + b^2}$$

Differentiate both sides:

$$dc = \frac{1}{2}(a^2 + b^2)^{-1/2} \cdot 2b db = \frac{b}{\sqrt{a^2 + b^2}} \cdot db$$

Substitute known values:

$$a = 5, \quad b = 12, \quad db = 0.1$$
$$c = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

$$dc = \frac{12}{13} \cdot 0.1 \approx \boxed{0.0923}$$

(a) **Maximum error:**

$$\boxed{dc \approx 0.0923 \text{ cm}}$$

(b) **Relative and percentage error:**

$$\text{Relative error} = \frac{dc}{c} = \frac{0.0923}{13} \approx 0.0071$$

$$\text{Percentage error} = 0.0071 \times 100 \approx \boxed{0.71\%}$$

Final Answers:

- Maximum error in hypotenuse: $\boxed{0.0923 \text{ cm}}$
- Relative error: $\boxed{0.0071}$
- Percentage error: $\boxed{0.71\%}$