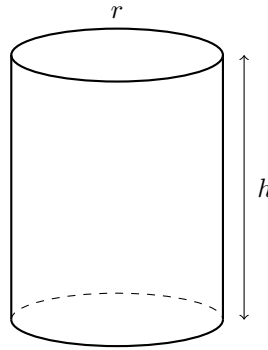


Question 6

The radius r of a right circular cylinder is measured to be 5 cm with a possible error of 0.1 cm. The height h is 20 cm with a possible error of 0.2 cm.

- (a) Use differentials to approximate the maximum error in calculating the volume of the cylinder.
- (b) Find the relative and percentage errors in the volume.



Question 6 - Solution

The volume of a cylinder is:

$$V = \pi r^2 h$$

We compute the differential dV :

$$dV = \frac{\partial V}{\partial r} dr + \frac{\partial V}{\partial h} dh = 2\pi r h dr + \pi r^2 dh$$

Given:

$$r = 5 \text{ cm}, \quad dr = 0.1 \text{ cm}, \quad h = 20 \text{ cm}, \quad dh = 0.2 \text{ cm}$$

Substitute into the differential:

$$dV = 2\pi(5)(20)(0.1) + \pi(5)^2(0.2) = 20\pi + 5\pi = \boxed{25\pi \approx 78.54 \text{ cm}^3}$$

(a) Maximum error:

$$\boxed{dV \approx 78.54 \text{ cm}^3}$$

(b) Relative and percentage error:

Actual volume:

$$V = \pi(5)^2(20) = 500\pi \approx 1570.8 \text{ cm}^3$$

Relative error:

$$\frac{dV}{V} = \frac{25\pi}{500\pi} = \frac{1}{20} = 0.05$$

Percentage error:

$$0.05 \times 100 = \boxed{5\%}$$

Final Answers:

- Maximum error in volume: $\boxed{25\pi \approx 78.54 \text{ cm}^3}$
- Relative error: $\boxed{0.05}$
- Percentage error: $\boxed{5\%}$