

Question 2

Suppose the radius r of a sphere is measured to be 5 cm with a possible error of 0.1 cm.

- (a) Use differentials to approximate the possible error in the calculated volume of the sphere.
- (b) Estimate the relative error and percentage error in the volume.

Question 2 - Solution

We use the volume formula of a sphere:

$$V = \frac{4}{3}\pi r^3$$

(a) **Approximate error using differentials:**

$$dV = \frac{dV}{dr} dr = 4\pi r^2 dr$$

$$dV = 4\pi(5)^2(0.1) = 4\pi(25)(0.1) = 10\pi \approx 31.42 \text{ cm}^3$$

(b) **Relative error:**

$$\frac{dV}{V} = \frac{10\pi}{\frac{4}{3}\pi(5)^3} = \frac{10\pi}{\frac{500}{3}\pi} = \frac{10 \cdot 3}{500} = \frac{30}{500} = 0.06$$

Percentage error:

$$0.06 \times 100 = \boxed{6\%}$$

Final Answer:

- Approximate error in volume: $\boxed{31.42 \text{ cm}^3}$
- Relative error: $\boxed{0.06}$
- Percentage error: $\boxed{6\%}$