

Question 1

Let $f(x) = x^3$.

- (a) Use differentials to approximate the change in f as x changes from 2 to 2.05.
- (b) Compare your result to the actual change $f(2.05) - f(2)$.

Question 1 - Solution

We are given:

$$f(x) = x^3, \quad f'(x) = 3x^2$$

(a) Use:

$$df = f'(x) dx$$

Let $x = 2$, $dx = 0.05$:

$$f'(2) = 3(2)^2 = 12$$

$$df = 12 \cdot 0.05 = \boxed{0.6}$$

So, the approximate change in f is $\boxed{0.6}$

(b) Actual change:

$$f(2.05) = (2.05)^3 = 8.615125, \quad f(2) = 8$$

$$\text{Actual change} = 8.615125 - 8 = \boxed{0.615125}$$

Conclusion: The differential estimate 0.6 is close to the actual change 0.615125