

**Question 10**

Use a linear approximation to estimate  $\sqrt{10.2}$ .

- (a) Let  $f(x) = \sqrt{x}$ . Choose a value  $a$  near 10.2 where the square root is easy to compute.
- (b) Find the linear approximation  $L(x)$  at  $x = a$ .
- (c) Use the approximation to estimate  $\sqrt{10.2}$  and compare with the actual value.

**Question 10 - Solution**

For  $f(x) = \sqrt{x}$ , use the nearby perfect square  $a = 9$ .

$$f(9) = 3, \quad f'(9) = \frac{1}{6}, \quad L(x) = 3 + \frac{x-9}{6}.$$

Therefore

$$\boxed{\sqrt{10.2} \approx L(10.2) = 3.2.}$$

Since  $f''(x) = -1/(4x^{3/2}) < 0$  for  $x > 0$ , the tangent line lies above the curve, so this is an overestimate. For comparison,  $\sqrt{10.2} \approx 3.193743885$ ; the error is approximately 0.006256115.