

Question 9

Use a linear approximation to estimate $\ln(1.05)$.

- (a) Define a function $f(x)$ such that $f(x) = \ln(x)$, and choose a value a near 1.05 where $f(a)$ is easy to compute.
- (b) Find the linear approximation $L(x)$ at $x = a$.
- (c) Use the approximation to estimate $\ln(1.05)$ and compare with the actual value.

Question 9 - Solution

(a) Let $f(x) = \ln(x)$, and choose $a = 1$ since $\ln(1) = 0$ is easy to compute.

$$f(a) = \ln(1) = 0, \quad f'(x) = \frac{1}{x}, \quad f'(1) = 1$$

(b) Linear approximation:

$$L(x) = f(a) + f'(a)(x - a) = 0 + 1(x - 1) = x - 1$$

(c) Estimate $\ln(1.05)$ using:

$$L(1.05) = 1.05 - 1 = 0.05$$

Actual: $\ln(1.05) \approx 0.04879$

Conclusion:

$$\boxed{\ln(1.05) \approx 0.05} \quad (\text{Linear Approximation})$$