

Question 8

Use a linear approximation to estimate $\sqrt{10.2}$.

- (a) Define an appropriate function $f(x)$ and choose a point a close to 10.2 where $f(a)$ is easy to compute.
- (b) Find the linear approximation $L(x)$ to $f(x)$ at $x = a$.
- (c) Use the approximation to estimate $\sqrt{10.2}$, and compare it with the actual value.

Question 8 - Solution

For $f(x) = \sqrt{x}$, use the nearby perfect square $a = 9$.

$$f(9) = 3, \quad f'(9) = \frac{1}{6}, \quad L(x) = 3 + \frac{x-9}{6}.$$

Therefore

$\sqrt{10.2} \approx L(10.2) = 3.2.$

Since $f''(x) = -1/(4x^{3/2}) < 0$ for $x > 0$, the tangent line lies above the curve, so this is an overestimate. For comparison, $\sqrt{10.2} \approx 3.193743885$; the error is approximately 0.006256115.