

Question 7

Use linear approximation to estimate $\ln(1.05)$.

- (a) Define a function $f(x)$ and choose a suitable point a near 1.05.
- (b) Find the linear approximation $L(x)$ of $f(x)$ at $x = a$.
- (c) Use $L(x)$ to estimate $\ln(1.05)$ and compare to the actual value.

Question 7 - Solution

Let $f(x) = \ln(x)$. We choose $a = 1$ since $\ln(1) = 0$ is easy to compute.

(a) Compute the derivative:

$$f'(x) = \frac{1}{x}, \quad \text{so } f'(1) = 1$$

(b) Linear approximation:

$$L(x) = f(a) + f'(a)(x - a) = 0 + 1(x - 1) = x - 1$$

(c) Estimate:

$$L(1.05) = 1.05 - 1 = 0.05$$

Compare to actual:

$$\ln(1.05) \approx 0.04879 \quad (\text{from calculator})$$

Conclusion:

$$\boxed{\ln(1.05) \approx 0.05} \quad (\text{Linear Approximation})$$

Actual: $\ln(1.05) \approx 0.04879 \Rightarrow$ Very close!