

Question 6

Use linear approximation to estimate $\sqrt[3]{8.2}$.

- (a) Define an appropriate function $f(x)$, and choose a point a near 8.2.
- (b) Find the linear approximation $L(x)$ of $f(x)$ at a .
- (c) Use $L(x)$ to estimate $\sqrt[3]{8.2}$, and compare it to the actual value.

Question 6 - Solution

Let $f(x) = \sqrt[3]{x} = x^{1/3}$, and choose $a = 8$ since $\sqrt[3]{8} = 2$ is exact and close to 8.2.

(a) Compute derivative:

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}, \quad \text{so} \quad f'(8) = \frac{1}{3\sqrt[3]{64}} = \frac{1}{3 \cdot 4} = \frac{1}{12}$$

(b) Linear approximation:

$$L(x) = f(a) + f'(a)(x - a) = 2 + \frac{1}{12}(x - 8)$$

(c) Estimate:

$$L(8.2) = 2 + \frac{1}{12}(0.2) = 2 + \frac{0.2}{12} = 2 + \frac{1}{60} \approx 2.0167$$

Actual value:

$$\sqrt[3]{8.2} \approx 2.0164 \quad (\text{from calculator})$$

Conclusion:

$$\boxed{\sqrt[3]{8.2} \approx 2.0167} \quad (\text{Linear Approximation})$$

$$\text{Actual: } \sqrt[3]{8.2} \approx 2.0164 \Rightarrow \text{Very close!}$$