

Question 5

Use linear approximation to estimate $\ln(1.05)$.

- (a) Define an appropriate function $f(x)$, and choose a convenient point a near 1.05.
- (b) Find the linear approximation $L(x)$ of $f(x)$ at a .
- (c) Use $L(x)$ to approximate $\ln(1.05)$, and compare it with the actual value.

Question 5 - Solution

Let $f(x) = \ln(x)$. Choose $a = 1$, since $\ln(1) = 0$ is simple and near 1.05.

(a) Derivative of $f(x)$:

$$f'(x) = \frac{1}{x}, \quad f'(1) = 1$$

(b) Linear approximation:

$$L(x) = f(a) + f'(a)(x - a) = \ln(1) + 1(x - 1) = x - 1$$

(c) Estimate:

$$L(1.05) = 1.05 - 1 = 0.05$$

Actual value:

$$\ln(1.05) \approx 0.04879 \quad (\text{using calculator})$$

Conclusion:

$$\boxed{\ln(1.05) \approx 0.05} \quad (\text{Linear Approximation})$$

Actual: $\ln(1.05) \approx 0.04879 \Rightarrow$ Very close!