

Question 4

Use a linear approximation to estimate $\sqrt[3]{8.1}$.

- (a) Identify a function $f(x)$, choose a suitable point a , and write the linear approximation $L(x)$.
- (b) Use $L(x)$ to estimate $\sqrt[3]{8.1}$.
- (c) Compare the estimate to the actual value.

Question 4 - Solution

Let $f(x) = \sqrt[3]{x} = x^{1/3}$, and choose $a = 8$, since $\sqrt[3]{8} = 2$.

(a) Linear approximation formula:

$$L(x) = f(a) + f'(a)(x - a)$$

Compute:

$$f'(x) = \frac{1}{3}x^{-2/3}, \quad f'(8) = \frac{1}{3} \cdot 8^{-2/3} = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$$

So:

$$L(x) = 2 + \frac{1}{12}(x - 8)$$

(b) Estimate:

$$L(8.1) = 2 + \frac{1}{12}(0.1) = 2 + \frac{1}{120} = \frac{241}{120} \approx 2.0083$$

(c) Actual value:

$$\sqrt[3]{8.1} \approx 2.00828 \quad (\text{using calculator})$$

Conclusion: The linear approximation gives:

$\sqrt[3]{8.1} \approx 2.0083$

which is very close to the actual value.