

Question 3

Use a linear approximation to estimate the value of $\ln(1.05)$.

- (a) Identify a function $f(x)$, choose a point a , and write the linear approximation $L(x)$.
- (b) Use $L(x)$ to estimate $\ln(1.05)$.
- (c) Compare the estimate to the actual value.

Question 3 - Solution

Let $f(x) = \ln(x)$, and choose $a = 1$ because $\ln(1) = 0$ is easy to compute.

(a) Linear approximation formula:

$$L(x) = f(a) + f'(a)(x - a)$$

Compute:

$$f(x) = \ln(x), \quad f'(x) = \frac{1}{x}, \quad f'(1) = 1$$

So:

$$L(x) = \ln(1) + 1 \cdot (x - 1) = 0 + (x - 1) = x - 1$$

(b) Estimate:

$$L(1.05) = 1.05 - 1 = 0.05$$

(c) Actual value:

$$\ln(1.05) \approx 0.04879 \quad (\text{using calculator})$$

Conclusion: The linear approximation gives:

$\ln(1.05) \approx 0.05$

which is a very good estimate.