

Question 2

Use a linear approximation to estimate the value of $\sqrt[3]{8.1}$.

- (a) Identify a function $f(x)$, a point a , and write the linear approximation $L(x)$.
- (b) Use $L(x)$ to estimate $\sqrt[3]{8.1}$.
- (c) Compare the estimate to the actual value using a calculator.

Question 2 - Solution

Let $f(x) = \sqrt[3]{x} = x^{1/3}$. We choose $a = 8$, since $\sqrt[3]{8} = 2$ is easy to compute.

(a) Linear approximation formula:

$$L(x) = f(a) + f'(a)(x - a)$$

Compute:

$$f(x) = x^{1/3}, \quad f'(x) = \frac{1}{3}x^{-2/3} \Rightarrow f'(8) = \frac{1}{3(8)^{2/3}} = \frac{1}{3 \cdot 4} = \frac{1}{12}$$

So:

$$L(x) = f(8) + f'(8)(x - 8) = 2 + \frac{1}{12}(x - 8)$$

(b) Estimate:

$$L(8.1) = 2 + \frac{1}{12}(8.1 - 8) = 2 + \frac{1}{12}(0.1) = 2 + \frac{1}{120} \approx 2.0083$$

(c) Actual value:

$$\sqrt[3]{8.1} \approx 2.0083 \quad (\text{using calculator})$$

Conclusion: The linear approximation gives a very close estimate:

$\sqrt[3]{8.1} \approx 2.0083$