

Question 1

Use a linear approximation to estimate $\sqrt{4.1}$.

- (a) Define the function and the point where you linearize.
- (b) Find the linearization $L(x)$ at the chosen point.
- (c) Use $L(x)$ to estimate $\sqrt{4.1}$.

Question 1 - Solution

(a) Let $f(x) = \sqrt{x}$

Choose point $a = 4$, since $\sqrt{4} = 2$ is easy to compute.

(b) Linearization formula:

$$L(x) = f(a) + f'(a)(x - a)$$

Compute derivative:

$$f'(x) = \frac{1}{2\sqrt{x}}, \quad f'(4) = \frac{1}{2 \cdot 2} = \frac{1}{4}$$

$$L(x) = 2 + \frac{1}{4}(x - 4)$$

(c) Estimate $\sqrt{4.1} \approx L(4.1)$:

$$L(4.1) = 2 + \frac{1}{4}(0.1) = 2 + 0.025 = \boxed{2.025}$$