

Question 10

Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{\sin(5x)}{\tan(3x)}$$

Question 10 - Solution

As $x \rightarrow 0$:

$$\sin(5x) \rightarrow 0, \quad \tan(3x) \rightarrow 0 \Rightarrow \text{Indeterminate form } \frac{0}{0}$$

Apply L'Hospital's Rule:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin(5x)}{\tan(3x)} &= \lim_{x \rightarrow 0} \frac{d}{dx} \left[\frac{\sin(5x)}{\tan(3x)} \right] = \lim_{x \rightarrow 0} \frac{5 \cos(5x)}{3 \sec^2(3x)} \\ &= \lim_{x \rightarrow 0} \frac{5 \cos(5x)}{3 \cdot \frac{1}{\cos^2(3x)}} = \lim_{x \rightarrow 0} \frac{5 \cos(5x) \cos^2(3x)}{3} \end{aligned}$$

As $x \rightarrow 0$:

$$\cos(5x) \rightarrow 1, \quad \cos(3x) \rightarrow 1 \Rightarrow \cos^2(3x) \rightarrow 1$$

So,

$$\frac{5 \cdot 1 \cdot 1}{3} = \frac{5}{3}$$

Final Answer:

$$\boxed{\frac{5}{3}}$$