

Question 8

Evaluate the limit:

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x)$$

Question 8 - Solution

As $x \rightarrow \infty$, we have:

$$\sqrt{x^2 + x} \rightarrow \infty, \quad x \rightarrow \infty \Rightarrow \text{Indeterminate form: } \infty - \infty$$

To resolve this, multiply by the conjugate:

$$\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x) &= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + x} - x)(\sqrt{x^2 + x} + x)}{\sqrt{x^2 + x} + x} \\ &= \lim_{x \rightarrow \infty} \frac{(x^2 + x) - x^2}{\sqrt{x^2 + x} + x} = \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + x} + x} \end{aligned}$$

Now simplify numerator and denominator:

Factor x from the square root:

$$= \lim_{x \rightarrow \infty} \frac{x}{x \left(\sqrt{1 + \frac{1}{x}} + 1 \right)} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{x}} + 1}$$

As $x \rightarrow \infty$, $\frac{1}{x} \rightarrow 0$, so:

$$\lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{x}} + 1} = \frac{1}{1 + 1} = \frac{1}{2}$$

Final Answer:

$$\boxed{\frac{1}{2}}$$