

Question 6

Evaluate the limit:

$$\lim_{x \rightarrow \infty} (x - \ln x)$$

Question 6 - Solution

As $x \rightarrow \infty$, we observe:

$$x \rightarrow \infty, \quad \ln x \rightarrow \infty$$

So the limit is of the form:

$$\infty - \infty \quad (\text{indeterminate form})$$

To apply L'Hospital's Rule, we rewrite the expression to form a quotient. Factor:

$$x - \ln x = x \left(1 - \frac{\ln x}{x} \right)$$

Now evaluate:

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0 \quad (\text{since exponential grows faster})$$

So:

$$\lim_{x \rightarrow \infty} (x - \ln x) = \infty \cdot (1 - 0) = \infty$$

Final Answer: ∞