

Question 4

Evaluate the limit:

$$\lim_{x \rightarrow 0^+} (1 + 2x)^{\frac{1}{x}}$$

Question 4 - Solution

We identify the limit as the indeterminate form:

$$\lim_{x \rightarrow 0^+} (1 + 2x)^{\frac{1}{x}} = 1^\infty$$

Let:

$$y = (1 + 2x)^{\frac{1}{x}} \Rightarrow \ln y = \frac{1}{x} \ln(1 + 2x)$$

Now compute the limit:

$$\lim_{x \rightarrow 0^+} \ln y = \lim_{x \rightarrow 0^+} \frac{\ln(1 + 2x)}{x}$$

This is a $\frac{0}{0}$ form. Apply L'Hospital's Rule:

$$= \lim_{x \rightarrow 0^+} \frac{\frac{d}{dx} \ln(1 + 2x)}{\frac{d}{dx} x} = \lim_{x \rightarrow 0^+} \frac{\frac{2}{1+2x}}{1} = \frac{2}{1} = 2$$

So:

$$\ln y \rightarrow 2 \quad \Rightarrow \quad y \rightarrow e^2$$

Final Answer:

$$\boxed{e^2}$$