

Question 2

Evaluate the limit:

$$\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$$

Question 2 - Solution

We observe that both $\frac{1}{x} \rightarrow \infty$ and $\frac{1}{\sin x} \rightarrow \infty$ as $x \rightarrow 0^+$, so the expression is of the indeterminate form:

$$\infty - \infty$$

We first combine into a single fraction:

$$\frac{1}{x} - \frac{1}{\sin x} = \frac{\sin x - x}{x \sin x}$$

Now the limit becomes:

$$\lim_{x \rightarrow 0^+} \frac{\sin x - x}{x \sin x}$$

As $x \rightarrow 0^+$, both numerator and denominator $\rightarrow 0$. This is now a $\frac{0}{0}$ indeterminate form, so we apply L'Hospital's Rule:

$$\lim_{x \rightarrow 0^+} \frac{\sin x - x}{x \sin x} = \lim_{x \rightarrow 0^+} \frac{\cos x - 1}{\sin x + x \cos x}$$

Now evaluate the new limit:

- Numerator: $\cos x - 1 \rightarrow 0$ - Denominator: $\sin x + x \cos x \rightarrow 0$

Still indeterminate! Apply L'Hospital's Rule again:

$$= \lim_{x \rightarrow 0^+} \frac{-\sin x}{\cos x + \cos x - x \sin x} = \lim_{x \rightarrow 0^+} \frac{-\sin x}{2 \cos x - x \sin x}$$

Now evaluate: - Numerator: $-\sin x \rightarrow 0$ - Denominator: $2 \cos x - x \sin x \rightarrow 2$

So the final result is:

$$\boxed{0}$$