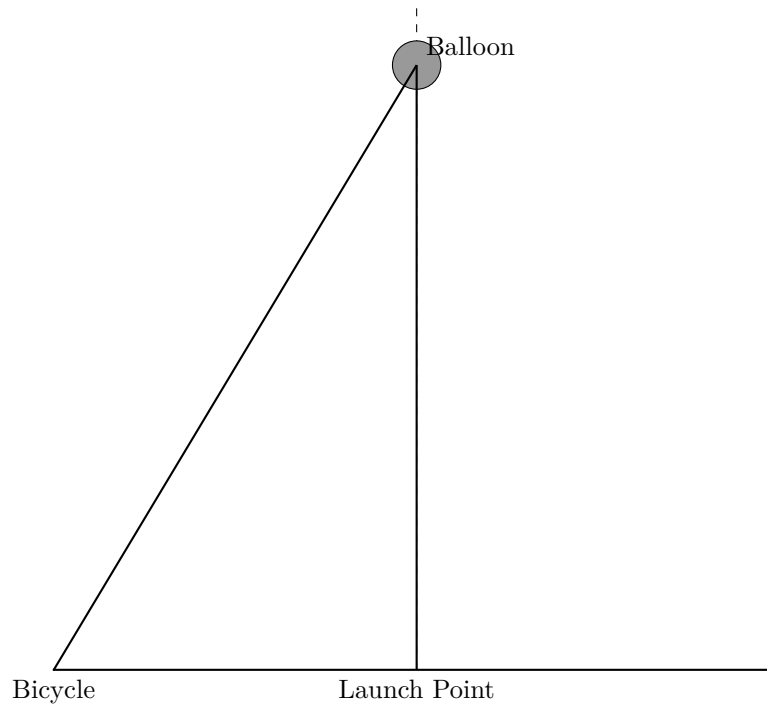


Question 10

Problem:

A hot air balloon is descending vertically at a rate of 2 ft/s, while a bicyclist passes directly beneath it, traveling in a straight line at a speed of 15 ft/s. How fast is the distance between the bicyclist and the balloon changing when the bicyclist is 60 feet from the point directly below the balloon and the balloon is 100 feet above the ground?



Question 10 - Solution

Let:

- $x(t)$: horizontal distance of bicycle from point below the balloon — $\frac{dx}{dt} = 15$
- $y(t)$: vertical distance of balloon from ground — $\frac{dy}{dt} = -2$
- $z(t)$: distance between balloon and bicycle

Using the Pythagorean Theorem:

$$z^2 = x^2 + y^2$$

Differentiate both sides with respect to time t :

$$2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt} \Rightarrow z \frac{dz}{dt} = x \frac{dx}{dt} + y \frac{dy}{dt}$$

Plug in known values:

$$x = 60, \quad y = 100, \quad z = \sqrt{60^2 + 100^2} = \sqrt{3600 + 10000} = \sqrt{13600} = 116.6$$

$$z \frac{dz}{dt} = 60(15) + 100(-2) = 900 - 200 = 700 \Rightarrow \frac{dz}{dt} = \frac{700}{116.6} \approx 6.01 \text{ ft/s}$$

Answer:

$$\boxed{\frac{dz}{dt} \approx 6.01 \text{ ft/s}}$$