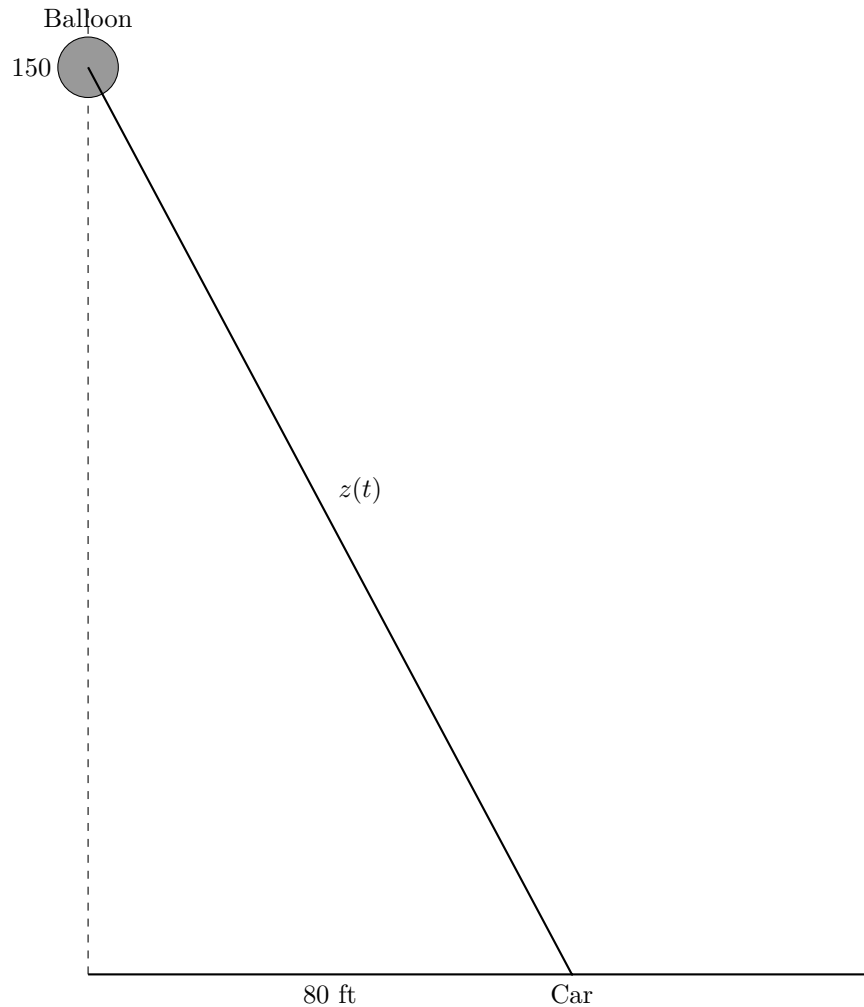


Question 9

Problem:

A balloon is rising vertically at a constant rate of 5 ft/s, and at the same time, a car drives directly away from the point on the ground beneath the balloon at 60 ft/s. How fast is the distance between the car and the balloon increasing when the car is 80 feet from the balloon's launch point and the balloon is 150 feet high?



Question 9 - Solution

Let:

- $x(t)$: distance from car to balloon launch point (horizontal) — $\frac{dx}{dt} = 60$
- $y(t)$: height of balloon — $\frac{dy}{dt} = 5$
- $z(t)$: distance between car and balloon (hypotenuse)

We apply the Pythagorean theorem:

$$z^2 = x^2 + y^2$$

Differentiate both sides:

$$2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt} \Rightarrow z \frac{dz}{dt} = x \frac{dx}{dt} + y \frac{dy}{dt}$$

Plug in values:

$$x = 80, \quad y = 150 \Rightarrow z = \sqrt{80^2 + 150^2} = \sqrt{6400 + 22500} = \sqrt{28900} = 170$$

Now plug in:

$$170 \cdot \frac{dz}{dt} = 80 \cdot 60 + 150 \cdot 5 \Rightarrow 170 \cdot \frac{dz}{dt} = 4800 + 750 = 5550 \Rightarrow \frac{dz}{dt} = \frac{5550}{170} = 32.65 \text{ ft/s}$$

Answer:

$\frac{dz}{dt} = 32.65 \text{ ft/s}$
