

## Question 5

**Lighthouse Problem:**

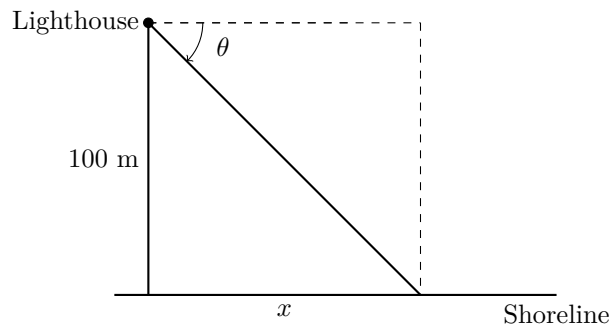
A lighthouse is located at the top of a cliff that is 100 meters high. The shoreline is straight and lies in a vertical plane. The beam of light rotates in this plane at a constant rate of

$$\frac{d\theta}{dt} = 0.05 \text{ rad/s.}$$

Let  $x$  be the distance along the shoreline from the base of the cliff to the point where the light hits the shore.

How fast is the spot of light moving along the shoreline when

$$\theta = \frac{\pi}{4} ?$$



**Question 5 - Solution**

Let:

- $\theta$  be the angle between the light beam and the horizontal shoreline.
- $x$  be the distance along the shoreline to the spot of light.

From the right triangle formed by the beam, the cliff, and the shoreline:

$$\tan(\theta) = \frac{x}{100} \quad \Rightarrow \quad x = 100 \tan(\theta)$$

Differentiate both sides with respect to time  $t$ :

$$\frac{dx}{dt} = 100 \sec^2(\theta) \frac{d\theta}{dt}$$

Substitute the given values:

$$\theta = \frac{\pi}{4}, \quad \frac{d\theta}{dt} = 0.05 \text{ rad/s}, \quad \sec^2\left(\frac{\pi}{4}\right) = 2$$

$$\frac{dx}{dt} = 100(2)(0.05) = 10 \text{ m/s}$$

**Answer:**

$$\frac{dx}{dt} = 10 \text{ m/s}$$